A

1/ $m = \lambda \ell$

2/ $I_{Oz} = \int dm\, r^2 = \int \lambda\, dr\, r^2 = \lambda \int_0^\ell r^2 dr = \lambda \frac{\ell^3}{3} = \frac{m\ell^2}{3}$

3/ $\vec{\Omega} = \dot{\theta}\, \vec{e}_3$

4/ $\vec{L}_O = \int_0^\ell \vec{OM} \wedge dm\, \vec{v}(M) = \int_0^\ell r \vec{e}_r \wedge \lambda\, dr\, r\dot{\theta}\, \vec{e}_\theta = \lambda \dot{\theta} \vec{e}_3 \int_0^\ell r^2 dr$

$\vec{L}_O = \lambda \dot{\theta} \frac{\ell^3}{3} \vec{e}_3 = m \frac{\ell^2}{3} \dot{\theta} \vec{e}_3 = I_{Oz} \vec{\Omega}$ avec $I_{Oz} = \frac{m\ell^2}{3}$

5/ $\vec{OG}$ | $\frac{\ell}{2}\cos\theta$; $\frac{\ell}{2}\sin\theta$; 0

6/ $\vec{v}_G$ | $-\frac{\ell}{2}\dot{\theta}\sin\theta$; $\frac{\ell}{2}\dot{\theta}\cos\theta$; 0

7/ $I_\Delta = \lambda \int_{-\ell/2}^{\ell/2} dr\, r^2 = \frac{\lambda}{3} 2 \left(\frac{\ell}{2}\right)^3 = \frac{\lambda \ell^3}{12} = \frac{m\ell^2}{12}$

8/ $I_{Oz} = I_\Delta + md^2 = I_\Delta + m\left(\frac{\ell}{2}\right)^2 \Rightarrow I_\Delta = I_{Oz} - \frac{m\ell^2}{4} = m\ell^2\left(\frac{1}{3} - \frac{1}{4}\right)$

$I_\Delta = \frac{m\ell}{12}$

9/ $\vec{L}_O = \vec{L}^* + \vec{OG} \wedge m\vec{v}_G \Rightarrow \vec{L}^* = \vec{L}_O - m\vec{OG} \wedge \vec{v}_G$

$\vec{L}^* = m\frac{\ell^2}{3}\dot{\theta}\vec{e}_3 - m\left[\left(\frac{\ell}{2}\right)^2 \dot{\theta}\cos^2\theta + \left(\frac{\ell}{2}\right)^2 \dot{\theta}\sin^2\theta\right]\vec{e}_3 = \frac{m\ell^2}{12}\dot{\theta}\vec{e}_3 = I_\Delta \vec{\Omega}$

10/ On retrouve $I_\Delta = m\ell^2/12$

13 11/ $I_\Sigma = \int_0^R \sigma\, 2\pi r\, dr\, r^2 = 2\pi\sigma \frac{R^4}{4} = \pi\sigma\frac{R^4}{2} = \frac{mR^2}{2}$ car $m = \sigma\pi R^2$

12/ $\vec{\Omega} = \dot{\theta}\vec{e}_3$

13/ $\vec{v}_{\Sigma_2} = \vec{0}$

14/ $\vec{v}_{\Sigma_1} = \vec{0}$

15/ $\vec{0} = \vec{v}_G + \vec{\Omega} \wedge G\vec{I} = \dot{x}\vec{e}_x + \dot{\theta}\vec{e}_3 \wedge (-R\vec{e}_y)$

$= (\dot{x} + R\dot{\theta})\vec{e}_x = \vec{0} \Rightarrow \dot{x} + R\dot{\theta} = 0$

16/ Signe $\oplus$ en effet $dx = -R\,d\theta$ le disque recule lorsque $\theta \uparrow$.